

# Search for the Heisenberg spin glass on rewired square lattices with antiferromagnetic interaction

Tasrief Surungan,<sup>\*</sup> Bansawang BJ,<sup>†</sup> and Dahlang Tahir<sup>‡</sup>

*Department of Physics, Hasanuddin University, Makassar, South Sulawesi 90245, Indonesia*

Spin glass (SG) is a typical magnetic system with frozen random spin orientation at low temperatures. The system exhibits rich physical properties, such as infinite number of ground states, memory effect and aging phenomena. There are two main ingredients considered to be pivotal for the existence of SG behavior, namely, frustration and randomness. For the canonical SG system, frustration is led by the presence of competing interaction between ferromagnetic (FM) and antiferromagnetic (AF) couplings. Previously, Bartolozzi *et al.* [Phys. Rev. **B73**, 224419 (2006)], reported the SG properties of the AF Ising spins on scale free network (SFN). It is a new type of SG, different from the canonical one which requires the presence of both FM and AF couplings. In this new system, frustration is purely caused by the topological factor and its randomness is related to the irregular connectivity. Recently, Surungan *et al.* [Journal of Physics: Conference Series 640, 012001 (2015)] reported SG behavior of AF Heisenberg model on SFN. We further investigate this type of system by studying an AF Heisenberg model on rewired square lattices. We used Replica Exchange algorithm of Monte Carlo Method and calculated the SG order parameter to search for the existence of SG phase.

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## I. INTRODUCTION

The study of Spin glasses has been an active research field for almost four decades [1–4]. It is a random magnetic system which is mainly characterized by a frozen random spins configuration at low temperature. This type of system has no total magnetization at any temperature, thus does not behave like a regular magnet. It is put in the group of magnetic system due to the cooperative phenomena of the spins, whose low temperature phase can still be regarded as ordered phase, i.e., a temporally ordered phase rather than spatially ordered phase [5]. The spin glass phenomenon was first experimentally reported in the early 70s by Cannella and Mydosh who observed the presence of a cusp instead of sharp peak of ac-susceptibility [1] of transition metal impurities hosted by noble metals (*CuMn* and *AuFe*). These are magnetic alloys with a small amount of magnetic impurities (Mn and Fe) randomly substituted into the lattice of non-magnetic hosts (*Cu* and *Au*).

It has been well understood that there are two main ingredients responsible for the existence of spin glass phase, namely frustration and randomness. A spin is frustrated if it can not find satisfactory orientation in interacting with its neighbor spins for minimizing the free energy. This can be exemplified by spins in a unit of triangular lattice with antiferromagnetic (AF) interactions; or by spins in a plaquette where both ferromagnetic (FM) and AF couplings exist. Spins interacting ferromagnet-

ically (anti-ferromagnetically) will prefer to be aligned (anti-aligned) to minimize the free energy. Therefore, for a plaquette to render a frustration, it has to fulfill a certain requirement, namely the product of all existing coupling interactions has to be negative. The FM and AF couplings are usually assigned by +1 and −1, respectively. According to this rule, a plaquette with positive product of couplings can not lead to a frustration as all spins are happy to each other, in the sense that there is no conflicting orientation.

Earlier theoretical studies of spin glasses were performed mainly on Ising systems with infinite and finite range of interaction [2, 6–8]. Several important results were found, such as the replica symmetry breaking scenario, the existence of infinite number of pure equilibrium states and the ultrametric trees grouping the pure states, etc. Some real SG materials are indeed Ising systems. Nevertheless, many SG systems found experimentally, including the canonical ones, the first spin glass system observed, are Heisenberg type where spins are three dimensional vectors (O(3) symmetry).

There is an existing controversy on the existence of spin glass phase for three dimensional (3D) Heisenberg system. This is in contrast with the discrete spin cases, such as the Ising model where spin glass phase transition is observed. For Heisenberg model, finding the lower critical dimension  $d_l$ , below which there is no SG phase transition, has attracted a lot of interests. Coluzzi [9] studied Heisenberg spins for 4D case and observed SG phase transition. While several works [10, 11] reported the existence of Heisenberg SG in 3D, recent study by Kawamura and Nishikawa [3] reported that only a chiral glass exists and no spin glass phase found. These various results indicate that more elaborations on the mechanism of the emergence of SG phenomena are required.

<sup>\*</sup>Electronic address: tasrief@unhas.ac.id

<sup>†</sup>Electronic address: bansawang@science.unhas.ac.id

<sup>‡</sup>Electronic address: dahlang@science.unhas.ac.id

Motivated by recent study of Heisenberg SG on scale free network (SFN) [12], here we study a slightly different systems, namely a rewired 2D lattices with antiferromagnetic interaction. Both systems are purely anti-AF and have irregular connectivities. Each spin in the two systems may have different number of neighbors. However, the connectivity structures of the two are different as no nodes in the rewired lattice acting as a hub like in SFN. The rewired 2D lattice does not have small-world behaviour either, therefore the notion of regular lattice still remains. With respect to SG ingredients, both systems belong to a rather new SG model as their randomness and the frustration are rendered by factors different from that of the canonical system. As known, randomness in canonical SG systems, is related to the random distribution of FM and AF couplings and the frustration is due to the presence of competing interaction between them. This new type of model inherits the main ingredient of SGs as it consists of AF triangular units where spins are frustrated. In fact, the AF Ising model on SFN has been studied, in which SG phase was observed [13].

The main objective of the current study is to verify that irregular connectivity together with frustration induced by topological factor can also built up spin glass system. In addition, it is an alternative approach for resolving the existing controversy in 3D Heisenberg SG model. A randomly rewired 2D lattice can be constructed with either fully AF interactions or mixture of AF and FM, as well as with various coordination number. In this study we put an extra link to each site of the square lattice so that the average neighbours of each spin is 3.0. It is a depleted triangular lattice which is partially frustrated if all couplings are AF. The fully-frustrated (FF) system without randomness does not have SG transition, instead it has an ordered magnetic phase at low temperature. An example of this is FF clock model on triangular lattice [14]. In this piece of work, we will probe whether the AF Heisenberg spins on rewired 2D lattice exhibits spin glass phase transition.

We use the replica exchange algorithm [15] of the Monte Carlo method, we calculate the order parameters of spin glass behaviour, the so-called overlap parameter and its distribution. For an accurate determination of the critical temperature, we also evaluate the Binder parameter. The paper is organized as follows: Section 2 describes the model and the method. The results are discussed in Section 3. Section 4 is devoted for a summary and concluding remarks.

## II. MODEL AND METHOD OF SIMULATION

The Heisenberg model on a rewired 2D lattices can be written with the following Hamiltonian,

$$H = \sum_{\langle ij \rangle} \vec{s}_i \cdot \vec{s}_j \quad (1)$$

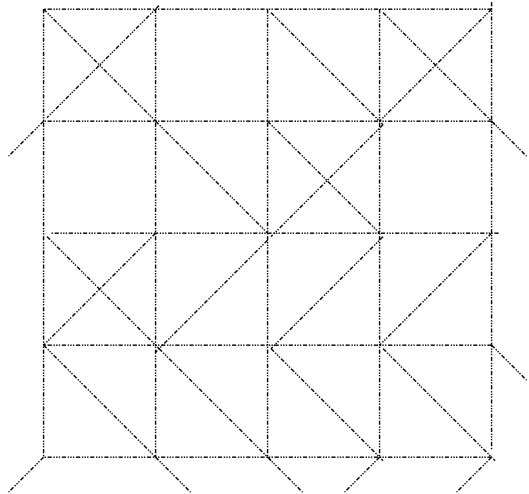


FIG. 1: The 2D rewired square lattice, where some sites have 7 links, others have 6 and the rest have 5. We consider periodic boundary condition.

where  $\vec{s}_i$  and  $\vec{s}_j$  are 3D vector spins, each occupying a site of the lattice. The summation is performed over all directly connected spins. In a regular lattice such as the square lattice, all spins have the same number of neighbours, associated to the integer coordination number. Here, we add one extra link to each site and randomly connect it to one of its next-nearest neighbors. Therefore, there are some sites having seven links, other having six and the rest only five. One particular realization of the lattice is shown in Fig. 1.

By excluding the double counting, the average number of links (ANOL) is 2.5. The lattice system can be regarded as a quasi regular lattice and in principle we can generate a lattice system with any fractional ANOL. As shown in the figure, the lattice consists of a large number of triangular units. Spins on each triangular unit are frustrated. Therefore, adding extra links corresponds to increasing the degree of frustration. In principle, it is possible to define the degree of frustration and study its contribution to the properties of the system, in particular the SG behaviour.

We performed Monte Carlo simulation using the replica exchange algorithm [15] to calculate thermal averages of the physical quantities of interest. This algorithm is particularly well to overcome the slow dynamics due to the existence of local minima in the energy landscape. The slow dynamics is a common problem in dealing with complex systems such as SGs where the random walker tend to be trapped at certain local minimum. It is an extended Metropolis algorithm where  $M$  replicas are calculated in parallel. Each replica is in equilibrium with a heat bath of inverse temperature. Given a set of the inverse temperatures,  $\beta$ , the probability distribution of finding the whole system in a state

$\{X\} = \{X_1, X_2, \dots, X_M\}$  is given by,

$$P(\{X, \beta\}) = \prod_{m=1}^M \tilde{P}(X_m, \beta_m), \quad (2)$$

with

$$\tilde{P}(X_m, \beta_m) = Z(\beta_m)^{-1} \exp(-\beta_m H(X_m)), \quad (3)$$

and  $Z(\beta_m)$  is the partition function for the  $m$ -th replica. We can then define an exchange matrix between replicas,  $W(X_m, \beta_m | X_n, \beta_n)$ , which is the probability to switch the configuration  $X_m$  at the temperature  $\beta_m$  with the configuration  $X_n$  at  $\beta_n$ . With the requirement to keep the entire system at equilibrium, we use the detailed balance condition on the transition matrix

$$P(\{X_m, \beta_m\}, \dots, \{X_n, \beta_n\}, \dots) \cdot W(X_m, \beta_m | X_n, \beta_n) = P(\{X_n, \beta_n\}, \dots, \{X_m, \beta_m\}, \dots) \cdot W(X_n, \beta_n | X_m, \beta_m), \quad (4)$$

along with Eq. (3), so that we have

$$\frac{W(X_m, \beta_m | X_n, \beta_n)}{W(X_n, \beta_n | X_m, \beta_m)} = \exp(-\Delta), \quad (5)$$

where  $\Delta = (\beta_n - \beta_m)(H(X_m) - H(X_n))$ . With this constraint, we can choose the matrix coefficients according to the standard Metropolis method which gives the following

$$W(X_m, \beta_m | X_n, \beta_n) = \begin{cases} 1 & \text{if } \Delta < 0, \\ \exp(-\Delta) & \text{if } \Delta > 0. \end{cases} \quad (6)$$

Due to the fact that the acceptance ratio decays exponentially with  $(\beta_n - \beta_m)$ , we restrict the exchange temperatures next to each other, i.e., the terms  $W(X_m, \beta_m | X_{m+1}, \beta_{m+1})$ . The replica exchange algorithm has been widely implemented in the study of various spin glass systems, including the AF-SFN Ising model where spin glass observed. In the next section, we present the results of our study.

### III. RESULTS AND DISCUSSION

#### A. Energy and the specific heat

We have simulated AF Heisenberg model on rewired square lattice of several systems with linear sizes  $L=32$ , 48, and 64. We implemented periodic boundary condition so that each site of the native square lattice has four neighbours. For each system size, we took many realizations of the lattice, then average the results over the number of realizations. This is a standard procedure in probing random systems such as SG. Each realization corresponds to one particular connectivity distribution which is randomly generated. For the results to be reliable, we have to take large number of realizations. Previous study of Heisenberg SG on SFN we took 1000 realizations. Here We took smaller number of realizations

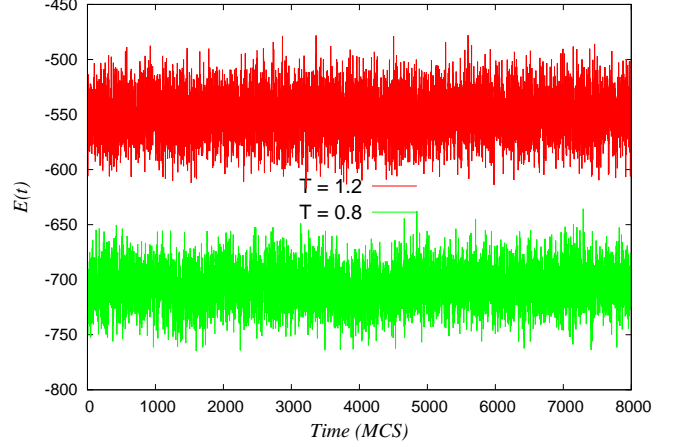


FIG. 2: Time series of energy for linear size  $L=32$  at temperature  $T = 1.2$  and  $T = 0.8$ .

( $N=250$ ) due to its degree of randomness is less compared to the previous system.

To check the reliability of our simulation we evaluated the energy time series of the system, from which we can obtain the average energy and specific heat. One can tell whether the system is in equilibrium or not by analysing the energy time series. In Monte Carlo simulation, time corresponds to a series of MCSs. One MCS is defined as one loop of updating each spin in the lattice, based on its probability. We perform  $M$  MCSs for each temperature and take  $N$  samples out of  $M$ . To make sure the system is well equilibrated we perform enough initial MCSs, usually 10 MCSs, before doing measurement.

The time series plot of energy for two different temperatures, i.e.,  $T=1.2$  and  $0.8$ , for linear size  $L = 32$  is shown in Fig. 2. This plot indicates that the system is well equilibrated after enough initial MCS, here it was taken 6000 MCSs. The fluctuation at higher temperature is larger due to larger thermal fluctuation. We extracted two quantities from energy time series, namely the ensemble average of energy,  $\langle E \rangle = \frac{1}{N} \sum_N E_i$ , and the specific heat which is defined as follows

$$C_v = \frac{N}{kT^2} (\langle E^2 \rangle - \langle E \rangle^2) \quad (7)$$

where  $N$  and  $k$  are respectively the number of spins and Boltzmann constant. The plot of these quantities are shown in Fig. 3. The specific heat plot has no a clear peak at finite temperature, which may signify that the SG transition is found only at  $T = 0$ . To clarify this we calculate the SG order parameter which is presented in the next sub-section.

#### B. Spin Glass Order Parameter

To search for the existence of SG phase transition, we calculate the overlapping parameter, also called as SG

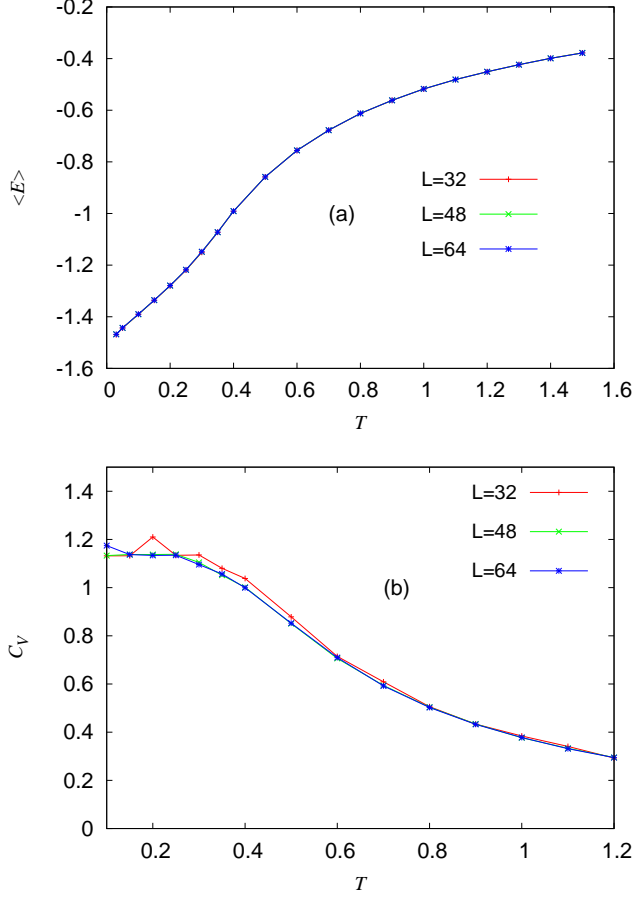


FIG. 3: Temperature dependence of Average energy (a) and specific heat (b) of three different sizes,  $L = 32, 48$  and  $64$ .

order parameter defined as follows

$$q_{EA} = \left\langle \left| \sum_i \vec{s}_i^\alpha \otimes \vec{s}_i^\beta \right| \right\rangle_{av} \quad (8)$$

This quantity basically originated from the scalar product of the vector spins. A scalar product of two vectors will give maximum value if they are parallel. If system is frozen, their overlapping parameter will give finite value. In the language of Ising spin, each term of this quantity is the overlapping of two possible states (up or down). For Heisenberg case, the overlapping of two spins is the dot product two 3D vectors. Because we have to accommodate the condition where vector spins may rotate in any direction, we take the tensor product instead of dot product, resulting in nine components of the quantity.

The plot of temperature dependence of  $q_{EA}$  for three different sizes,  $L = 32, 48$  and  $64$  is shown in Fig. 4. As indicated, the order parameter is increasing as temperature decreases, e.g., for  $L = 32$ ,  $q_{EA} > 0$  at  $T < 0.4$ .

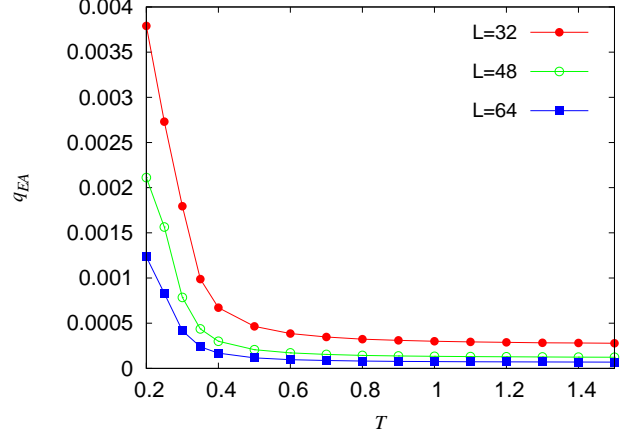


FIG. 4: Temperature dependence of SG order parameter of four different sizes,  $L = 32, 48$  and  $64$ .

However, as system sizes become larger, the value of  $q_{EA}$  tend to decrease. This is an indication that the parameter goes to zero at infinite value of  $L$ , which is a sign of the absence of SG phase at the thermodynamic limit. The result is consistent with the plot of specific heat which exhibits that SG phase transition occurs at zero temperature, in other words no finite temperature SG phase observed. The study of the model for various connectivity densities is still in progress. The results will be reported elsewhere.

#### IV. SUMMARY AND CONCLUSION

In summary, we have studied the AF Heisenberg spins on rewired square lattices and searched for the existence of SG phase. One extra link is added to each site of the lattices; which randomly connects the site to one of its next-nearest neighbors. The system is randomly frustrated due to the existence of abundance of triangular units. It is a new type of SG model which inherits the main ingredients of SGs, i.e., randomness and frustration. By using Replica Exchange Monte Carlo method, which is a standard method in SG study, we calculated several physical quantities, such ensemble average of energy, the specific heat and the overlapping parameter. We observed no finite temperature spin glass phase transition. This result suggested that the AF rewired 2D lattice with average connection 3.0 can not host the existence of SG phase. System with such average connectivity may correspond to the canonical models below the critical dimension. Further consideration of rewired 2D lattice with larger connectivity densities is required.

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